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A Mathematical Analysis of Centipede Game Theory in Emergency Room Interactions

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ABSTRACT ***Original research paper***

This paper explores physician-nurse interactions in emergency rooms using Centipede Game Theory, a sequential decision-making model. The game illustrates how trust and institutional factors influence cooperation under high-stakes conditions. Participants alternate between cooperating, which increases cumulative rewards, and defecting, which ends the interaction for immediate personal gain. We introduce a trust-sensitive version of the model, featuring a logistic cooperation probability function, PC (t), driven by trust cap (θ), sensitivity (λ), and inflection point (t0). The expected utility function, EUC (t), captures outcomes across ten stages. Using values $\delta = 1.2$, $\alpha = 0.6$, and $\theta = 0.8$, results show cooperation probability rising from 0.095 at stage 1 to 0.74 at stage 10, and EUC (t) increasing from 0.50 to 3.39. The findings underscore how trust and institutional support like joint training and standardized protocols can shift strategic behavior from early defection to sustained collaboration. This research demonstrates the value of game-theoretic approaches for improving teamwork and patient outcomes in critical healthcare environments.

1. Introduction

Decision-making in high-stakes clinical environments such as emergency rooms (ERs) often involve dynamic interactions between multiple healthcare professionals. Physicians and nurses must collaborate under time-sensitive and uncertain conditions to deliver optimal patient care. The efficiency and effectiveness of such collaboration are deeply influenced by institutional hierarchies, communication dynamics, trust levels, and incentive structures (Manojlovich & DeCicco, 2007; Zwarenstein et al., 2009). Enhancing the efficiency, quality, and long-term sustainability of the U.S. health care system is placing growing emphasis on the importance of generating stronger evidence for decision-making through comparative effectiveness research (Chalkidou et al., 2009). Decision-making is a fundamental aspect of daily human life. Since people make decisions of varying significance every day, the notion that decision-making can be challenging might appear unusual or even hard to believe (Li, 2008).

Among various game-theoretic models, the Centipede Game offers a compelling lens for understanding sequential decisions where players choose at each stage whether to cooperate or defect, with payoffs increasing the longer cooperation continues. Rosenthal (1981) paper argued that finite, noncooperative games with complete and perfect information should be analyzed similarly to singleplayer decision problems. Specifically, it suggested that at each stage of the game, players should assign subjective probabilities to future actions and make choices through backward induction. This perspective differs from the traditional game-theoretic framework based on Nash equilibrium. This structure mirrors real-world ER scenarios in which physicians and nurses must decide whether to continue collaborative patient management or act independently; often with differing short- and long-term consequences.

Okeke (2019) [6] work demonstrated the analytic of the convergence of an iterative sequence defined by a generalized Lipschitzian map on a cone metric space. Several studies have employed analytical methods alongside software tools such as SPSS and MATLAB for the analysis and interpretation of modelling results. For example, Okeke ([7]), Okeke & Akpan ([8]), and Okeke & Ifeoma ([9], [10]) examined various modelling approaches applied to physical phenomena and the sensitivity of coronavirus disparities in Nigeria. Okeke et al. (2019) [11] utilized analytical theorems, including the fixed point theorem. Okeke & Peters (2019) [12] emphasized numerical stability in physical flow applications using Software-assisted analysis tool (Okeke & Nwokolo, 2025 [13]). The principle of maximum was analytically applied to establish the uniqueness of solutions in metric spaces involving second-order linear Volterra integral equations.

Classical backward induction in game theory predicts early defection, as rational players attempt to maximize their individual payoffs by ending cooperation before their partner does. However, empirical studies have shown that real human behavior often deviates from this theoretical prediction, with sustained cooperation observed even in competitive environments (McKelvey & Palfrey, 1992; Johnson et al., 2002).

Trust emerges as a critical determinant in such deviations. In healthcare, trust not only enhances interprofessional communication but also directly correlates with improved patient outcomes (Hall et al., 2001). Yet, trust is neither static nor binary; it evolves over time and is sensitive to prior interactions, institutional culture, and perceived competence. To capture this dynamic quality, recent research integrates probabilistic trust functions into game-theoretic models, allowing for a more realistic representation of human decision-making over repeated interactions (Bacharach et al., 2007; Kimbrough & Vostroknutov, 2016).

This study introduces a modified Centipede Game framework tailored to emergency room dynamics, incorporating a logistic trust-based cooperation probability function. By applying a ten-stage sequential interaction model between a physician and a nurse, we examine how trust development influences strategic decisions at each stage. Parameters such as a collaboration multiplier (δ), trust cap (θ), and experience-based inflection point ($t0$) are mathematically modeled to simulate cooperation probability and expected utility. The outcomes illustrate how cooperation, though theoretically suboptimal in early stages, becomes increasingly dominant as trust builds, resulting in higher cumulative benefits for patient care.

Moreover, this modeling framework supports practical implications. By quantifying the conditions under which cooperation becomes the dominant strategy, it provides a rigorous basis for designing interventions such as team-based training, incentive alignment, and institutional protocol redesign that can shift ER dynamics toward more collaborative equilibria. Thus, mathematical game models, when integrated with behavioral and institutional parameters, offer powerful tools for understanding and improving the complex decision systems that underpin emergency healthcare delivery.

2. Review of Related Works

Understanding decision-making dynamics in high-pressure healthcare environments has long been a focus of interdisciplinary research, especially in emergency room (ER) settings where rapid, cooperative actions are essential. The application of game theory to healthcare has grown in prominence over the past two decades, providing a structured approach to analyze strategic interactions among professionals with potentially conflicting objectives. Early work by Levati et al. (2007) used experimental game theory to investigate trust and reciprocity in repeated interactions, laying the groundwork for modeling cooperation under uncertainty.

In the context of healthcare, authors have explored various game-theoretic frameworks to analyze coordination problems. For instance, Liu et al. (2016) applied evolutionary game theory to understand the cooperation between hospitals and patients under insurance incentives, while Zhang and Zhang (2020) examined Stackelberg games to optimize hospital resource allocation. However, few studies have addressed physician-nurse interactions specifically, despite their centrality in ER operations. These interactions often involve real-time decisions that require mutual trust and aligned incentives, making them well-suited for dynamic game-theoretic modeling.

The Centipede Game, introduced by Rosenthal (1981), provides a sequential framework for studying such trust-based decisions. It has been used to model cooperation in economics (McKelvey & Palfrey, 1992), behavioral psychology (Andreoni & Miller, 1993), and political science, but its application to healthcare team dynamics remains novel. In its standard form, the Centipede Game predicts early defection as the rational equilibrium, yet empirical evidence often shows sustained cooperation, particularly in repeated or institutionalized contexts. This divergence has prompted modifications incorporating bounded rationality, reputation effects, and, more recently, trust dynamics (Charness & Rabin, 2002).

Our work builds on these insights by integrating a logistic trust function into the Centipede framework, an approach that echoes methods in dynamic behavioral modeling (Fehr & Schmidt, 1999). By doing so, we reflect the evolving nature of trust in real-world clinical interactions, which are influenced by systemic factors like team training, shared protocols, and prior experiences (Reader et al., 2009). Studies in healthcare management also suggest that trust is not only interpersonal but deeply embedded in institutional practices (Weiner et al., 2008), reinforcing the importance of capturing both individual and systemic variables.

Furthermore, research on interprofessional collaboration in ERs emphasizes the impact of communication, hierarchy, and shared mental models on team performance (Manser, 2009; Kilner & Sheppard, 2010). These insights align with our model's emphasis on how structural trust such as collaborative routines can shift strategic equilibria toward cooperation. Although simulation-based studies have shown that team training can reduce medical errors and enhance coordination (Salas et al., 2008), a rigorous mathematical treatment of these dynamics remains underdeveloped. Our contribution addresses this gap by quantifying the effect of trust dynamics on cooperation over time in ER settings.

In summary, while existing literature has established the relevance of game theory, trust, and team dynamics in healthcare, our model uniquely combines these elements in a formal Centipede Game framework, providing a novel lens through which to examine and optimize real-time clinical decision-making.

This study strategically apply mathematical exploration using centipede game theory to solve problems related to emergency room interactions between the hospital workers such as physiciannurse. The objectives of this study include: to examine how trust and institutional factors influence cooperative behavior during high-stakes clinical decision-making; to determine the logistic cooperation probability function PC (t) that evolves with time and parameters such as trust cap θ, sensitivity λ, and inflection point $t0$; to study the expected utility function EUC (t) that integrates both cooperative and non-cooperative outcomes over ten stages; to plot the defective payoffs for both the physician and the nurse over the stages $t = 1$ to $t = 10$, assuming $\delta = 1.2$, $\alpha = 0.6$, and $\theta = 0.8$; and to show that while classical game theory predicts early defection, introducing trust-based probability functions and collaborative incentive structures can alter the equilibrium and promote sustained cooperation between physicians and nurses in emergency settings.

3. Methodology

The example of a Centipede theory Structure is given in the Figure 1 below. The Figure 1 illustrates a 10-stage centipede game modeled within a healthcare context, involving two hospitals, Hospital A and Hospital B. At each stage, the hospital whose turn it is must decide whether to Take — ending the cooperation to secure an immediate individual benefit — or Pass, allowing the collaboration to continue and potentially yield greater joint benefits. The payoffs at each terminal node show the distribution of benefits to both hospitals, with the first number representing Hospital A's payoff and the second Hospital B's.

As the game progresses, the potential rewards for cooperation increase, encouraging both hospitals to continue passing and collaborating. For example, early Take actions yield modest benefits like (2,1) or (1,3), while later cooperation can lead to higher payoffs such as (9,11). However, the temptation to defect and take the guaranteed immediate payoff is always present, representing the tension between short-term self-interest and long-term mutual gain.

This model reflects real-world healthcare decisions where hospitals must decide whether to share patient data, research findings, or resources. Cooperation can lead to improved patient outcomes and cost efficiencies, but distrust or fear of exploitation may cause premature withdrawal. Thus, the diagram highlights the strategic decision-making and risks inherent in collaborative healthcare partnerships. It emphasizes the importance of trust and the challenges in achieving sustained cooperation despite the potential for higher collective benefits.

This section also detailed Players, Game Structure and Payoff Function. Let there be two players:

i. P1: Physician
ii. P2 : Nurse

Assumption: They alternate turns to cooperate (C) or defect (D).

The game consists of n stages (decision points). At each stage t ∈ {1, 2, . . . , n}, the player whose turn it is chooses either:

i. C (Continue): Pass the decision to the next player and increase the total payoff.
ii. D (Defect): End the game and receive an immediate payoff. Where:

Table 1: The payoff structure, at stage t, δ > 1 and α, β ∈ (0, 1)

Choice1(C)	$v_t^{(1)}, v_t^{(2)} = \delta^t t \cdot \alpha, \delta^t \cdot (1-\alpha)$
Choice2(D)	$v_t^{(1)}, v_t^{(2)} = \delta^t t \cdot \beta, \delta^t \cdot (1-\beta)$

i. $\delta>1$:trustmultiplierorcollaborationgrowth rate
ii. $\alpha,\beta\in(0,1)$:proportionsofpayoffsplitdependingoncooperationordefection

3.1 Backward Induction (Subgame Perfect Nash Equilibrium), Modified Trust-Based Model and Expected Utility with Trust Factor

Using backward induction and following the Broome and Rabinowicz (1999), the rational strategy under classical game theory is:

At t = n : player defects to maximize payoff
At t = n – 1 : anticipating defection, the previous player also defects

⇒ Both players defect early (t = 1), resulting in suboptimal outcomes

The logistic growth function was introduced in a biological context by Pierre Fran¸ cois Verhulst in the 1830s containd in work of Miner (1933) when studying population dynamics. The general form representing the logistic growth is:

$$f(t) = \frac{L}{1 + e^{-k(t-t0)}} \qquad (1)$$

Where:

i. L is the carrying capacity (maximum value),
ii. k is the growth rate,
iii. t0 is the midpoint (inflection point),
iv. f(t) is the population or adoption level at time t.

This equation is widely used across many disciplines such as in population growth, epidemiology (disease spread), neural networks (activation function), economics (adoption of innovations), physics (phase transitions) and sociology (information diffusion).

3.2 Definitions of Terms

i. Trust Cap, θ ∈ (0, 1]: ˆ

- The maximum achievable level of cooperation under ideal circumstances. ˆ
- Represents the upper bound of the probability that an individual or agent will cooperate. ˆ
- A value of θ = 1 implies full trust is theoretically attainable, while lower values reflect systemic or structural limits to cooperation, even under favorable conditions.

ii. Sensitivity to Time or Experience, λ:

- Controls the rate at which cooperation increases over time. ˆ
- A higher λ implies that cooperation ramps up quickly with experience or exposure, indicating a highly responsive system.
- A lower λ results in a slower growth, suggesting that cooperation develops more gradually due to slower learning or institutional inertia.

iii. Inflection Point, t_0: ˆ

- The critical threshold or milestone at which the rate of change in cooperation is highest.
- Represents a turning point in experience or institutional development—such as completing a training program, reaching a policy milestone, or hitting a critical mass of exposure. ˆ

- Before t_0, cooperation increases slowly; after t_0, it accelerates, then gradually levels off toward θ.

  To incorporate trust and institutional protocols, we define a cooperation probability function from equation (1) as:

$$P_C(t) = \frac{\theta}{1 + e^{-\lambda(t-t_0)}} \qquad (2)$$

Where:

i. rust cap, $\theta \in (0, 1]$

ii. Sensitivity to time or experience, λ

iii. Inflection point (e.g., training milestone or experience threshold), t_0

The graph of $P_C(t) = \frac{\theta}{1+e^{-\lambda(t-t_0)}}$ is a sigmoid (S-shaped) curve used to model growth or transition processes. It starts near zero for very negative t, increases gradually, and sharply rises around the inflection point $t = t_0$, where the growth rate is maximal. As t increases further, the function levels off and asymptotically approaches the horizontal line $y = \theta$, representing the maximum value.

The curve is symmetric about $t = t_0$, and the value at the inflection point is $\frac{\theta}{2}$.

In mathematical exploration of the centipede game theory applied to emergency room (ER) interactions, the function $P_C(t) = \frac{\theta}{1+e^{-\lambda(t-t_0)}}$ represent the probability of cooperation between medical personnel over time. The sigmoid shape reflects how initial hesitation or mistrust (low t) gives way to rapid trust-building and collaboration as critical decisions arise near $t = t_0$, the inflection point. As time progresses and mutual benefits are recognized, cooperation stabilizes near θ, the maximum sustainable level. This aligns with game-theoretic predictions of increasing cooperation when future payoffs outweigh immediate self-interest.

Let the expected utility of cooperating at stage t be:

$$EU_C(t) = P_C(t) \cdot \delta^t \cdot \alpha + (1 - p_c(t)) \cdot v_t^{(1)} \qquad (3)$$

Players will choose to cooperate if:

$$EU_C(t) > v_t^{(1)} \quad \Rightarrow \text{ trust modifies the equilibrium}$$

Where:

i. EUC (t) is the expected utility at time t; it reflects the weighted average of outcomes when a player decides to cooperate,

ii. PC (t) is the probability that event C occurs at time t,

iii. δ t is the discount factor applied over time t,

iv. α is the utility received if event C occurs, v. v (1) t is the utility in the alternative scenario when event C does not occur.

3.3 Background Assumption

We are analyzing decision-making in a repeated game setting (such as the Centipede Game), where at each stage t, a player decides whether to:

i. **Cooperate** (trust the other player or institution), or

ii. **Defect** (act independently based on personal expected value).

Incorporating trust and institutional learning into the model means that the decision to cooperate is not deterministic, but instead depends on some probability of cooperation, PC (t), which evolves over time.

3.4 Defining Expected Utility of Cooperation, $EU_C(t)$

We assume:

i. A player cooperates at time t with probability $P_C(t)$.

ii. If cooperation succeeds, the player gains a discounted future payoff, represented by $\delta^t \cdot \alpha$, where:

iii. δ: Time discount factor (future rewards are worth less).

iv. α: Reward from successful cooperation (e.g., payoff from joint action).

v. If cooperation fails, the player receives a fallback or default utility, $v_t^{(1)}$, which is the payoff from unilateral action.

Thus, the expected utility of cooperation is given by:

$$EU_c(t) = \underbrace{P_c(t) \cdot \delta\, t \cdot \alpha}_{\text{Expected gain from cooperation}} + \underbrace{(1 - p_c(t)) \cdot (1 - PC(t)).v^{(1)}}_{\text{Fallback utility if cooperation fails}} \qquad 4$$

This is a probabilistic utility function that captures both trust and the reward structure.

3.5 Decision Rule: When Do Players Cooperate?

A rational player will choose to cooperate only if the expected utility from cooperating exceeds the utility from defecting:

$$EU_C(t) > V_t^{(1)} \qquad 5$$

Substituting EU_C (t):

$$p_c(t).\delta^t \cdot \alpha + (1 - p_c(t)) \cdot v_t^{(1)} > v_t^{(1)}$$

$$p_c(t).\left(\delta^t \cdot \alpha - v_t^{(1)}\right) > 0$$

So, cooperation is favorable only when the discounted cooperative payoff exceeds the fallback payoff:

$$\delta^t \cdot \alpha - v_t^{(1)} \qquad 6$$

and only to the extent that P_C (t) > 0; meaning trust modulates when this inequality holds.

3.6 Trust Modifies the Equilibrium

In this model:

i. The presence of a dynamic trust function PC (t), which increases over time or with institutional investment, shifts the equilibrium.

ii. Trust does not just reflect willingness to cooperate; it quantitatively alters the expected utility.

Thus:

$EU_C(t) > v_t^{(1)}$ ⇒ trust modifies the equilibrium

This is especially relevant in institutional or organizational settings where cooperation is learned or built over time (e.g., teams, hospitals, governments).

This structural formula is prevalent in several disciplines like behavioral economics and decision theory; models of intertemporal choice and prospect theory often use similar formulations; computational neuroscience and reinforcement learning; decision-making under uncertainty and over time is modeled. This has contributed meaningfully to the area; health economics and risk analysis; treatment decisions over time with uncertain outcomes.

4. Results

This section shows the computation of δ t at early stages and computation of coorperation probabilities, P_C (t).

4.1 Computation of δ t at Early Stages

Given the trust multiplier, δ = 1.1, then, the exponential growth of cooperation benefits at each stage t is computed as follows:

Table 2: The summary of the computed values oftrust multiplier, δ = 1.1 for each stage t from 1 to 10

Stage, t	1	2	3	4	5	6	7	8	9	10
δ^t	1.1	1.21	1.331	1.4641	1.61051	1.77156	1.94872	2.14359	2.35795	2.59374

4.2 Computation of Cooperation Probabilities $P_C(t)$:

θ = 0.95, λ = 0.6, t_0 = 2.573

The following table summarizes the computed values of PC (t) for each stage t from 1 to 10. This table shows how the cooperation probability increases with each stage t, reflecting the growing trust as the interaction progresses.

Table 3: The summary of the computed values of PC (t) for each stage t from 1 to 10

Stage, t	1	2	3	4	5	6	7	8	9	10
$P_C(t)$	0.266	0.394	0.536	0.667	0.770	0.842	0.888	0.915	0.930	0.939

We calculate the cooperative payoff for each stage t.

Table 4: Cooperative Payoff Table for t = 1 to t = 10, δ = 1.2, α = 0.6

Stage , t	Physician's Payoff $u_t^{(1)} = \delta^t \cdot \alpha$	Nurse's Payoff $u_t^{(2)} = \delta^t \cdot (1-\alpha)$
1	0.72	0.48
2	0.864	0.576
3	1.037	0.691
4	1.244	0.829
5	1.493	0.995
6	1.792	1.194
7	2.150	1.433
8	2.580	1.720
9	3.096	2.064
10	3.715	2.477

The Figure 3 below represents a 10-stage centipede game between two healthcare professionals: a Physician and a Nurse. Each stage models a decision point where the current player chooses either to Take the payoff immediately or Pass the opportunity to the other player, potentially increasing the joint benefits. The payoffs are based on the cooperative payoff table, calculated using the parameters δ=1.2 and α = 0.6 which reflect the growth factor and payoff distribution between the Physician and Nurse, respectively.

At early stages, the payoffs are relatively small, with the Physician receiving 0.72 and the Nurse 0.48 at stage 1. As the game progresses, the payoffs grow exponentially, reaching 3.715 for the Physician and 2.477 for the Nurse by stage 10. This increase incentivizes players to continue cooperating by passing their turn rather than taking the immediate payoff. The game captures the tension between short-term self-interest and long-term collaboration in healthcare settings, such as shared patient care or research. Both players must weigh the benefits of continuing cooperation against the risk of the other party defecting. The structure emphasizes the importance of trust and strategic decision-making in achieving optimal outcomes. Ultimately, the centipede game highlights how cooperation can yield greater collective benefits but requires patience and confidence in the other player's commitment.

Next, we calculate the Expected Utility with Trust Factor for each stage t = 1 to t = 10, assuming the following parameters. The expected utility is given by:

$$\mathrm{EU_C(t)} = P_C\,(\mathrm{t}) \cdot\ \delta^{\mathrm{t}} \cdot \alpha + (1 - \mathrm{p_c}\,(\mathrm{t})) \cdot \mathrm{v}_t^{(1)}$$

$$\text{where} \cdot \mathrm{v}_t^{(1)} = \delta^{\mathrm{t}} \cdot\ \beta.$$

Table 5: Expected Utility Table for t = 1 to t = 10, δ = 1.2, α = 0.6, θ = 0.8, λ = 0.5, t_0 = 5

Stage$t$	$P_C(t)$	$v_t^{(1)}$	$EU_C(t)$	Nature of Trust[$EU_C(t)>v_t^{(1)}$]
1	0.09536	0.48	0.50289	Equilibrium
2	0.14594	0.576	0.61803	Equilibrium
3	0.21515	0.6912	0.76556	Equilibrium
4	0.30203	0.829	0.95440	Equilibrium
5	0.40000	0.995	1.19420	Equilibrium
6	0.49797	1.194	1.49158	Equilibrium
7	0.58485	1.433	1.85228	Equilibrium
8	0.65406	1.720	2.28242	Equilibrium
9	0.70464	2.064	2.79110	Equilibrium
10	0.73931	2.477	3.39230	Equilibrium

We will plot the cooperative payoffs for both the physician and the nurse over the stages t = 1 to t = 10, assuming δ = 1.2 and α = 0.6.

The cooperative payoffs are given by:

$$(\mathrm{u}_{\mathrm{t}}^{(1)}, \mathrm{u}_{\mathrm{t}}^{(2)}) = \delta^{\mathrm{t}} \cdot \alpha, \delta^{\mathrm{t}} \cdot (1 - \alpha))$$

The graph in Figure 4 displays the cooperative payoffs for both the physician and the nurse over 10 stages. The blue curve represents the physician's increasing payoff, while the red curve shows the nurse's payoff. As the stages progress, both payoffs grow exponentially due to the trust multiplier δ.

4.3 Graphs of P_C (t), $\mathrm{v}_{\mathrm{t}}^{(1)}$, and $\mathrm{EU_C}$(t)

The graphs illustrate a continuous increase in cooperation probability P_C (t), value $\mathrm{v}_{\mathrm{t}}^{(1)}$, and expected utility $\mathrm{EU_C}$(t) from stages 1 to 10. As trust builds over time, P_C (t) rises sigmoidally, enhancing $\mathrm{EU_C}$(t), which consistently outpaces the base value $\mathrm{v}_{\mathrm{t}}^{(1)}$.

This table shows how the cooperation probability increases with each stage t, reflecting the growing trust as the interaction progresses.

Table 6: Simulated Values for Centipede Game Stages (Trust-Based Model) θ = 0.95, α = 0.6, β = 0.8, λ = 0.6, t0 = 2.573

Stage t	δ^t	$P_C(t)$	Coop. Payoff$(u_t^{(1)}, u_t^{(2)})$	Def. Payoff $(v_t^{(1)}, v_t^{(2)})$	$EU_C(t)$	Nature of Trust
1	1.100	0.26613	(0.660,0.440)	(0.880,0.220)	0.82145	No equilibrium
2	1.210	0.39414	(0.726,0.484)	(0.968,0.242)	0.87262	No equilibrium
3	1.331	0.53552	(0.7986,0.5324)	(1.0648,0.2662)	0.92224	No equilibrium
4	1.4641	0.66677	(0.87846,0.58564)	(1.17128,0.29282)	0.9760	No equilibrium
5	1.61051	0.77040	(0.96631,0.64420)	(1.28841,0.32210)	1.04026	No equilibrium
6	1.77156	0.84224	(1.06294,0.70862)	(1.417249,0.35431)	1.11883	No equilibrium
7	1.94871	0.88767	(1.16923,0.77949)	(1.55900,0.38974)	1.21301	No equilibrium
8	2.14359	0.91475	(1.28615,0.85744)	(1.714871,0.42872)	1.32270	No equilibrium
9	2.35795	0.93032	(1.41477,0.94318)	(1.88636,0.471590)	1.44763	No equilibrium
10	2.59374	0.93910	(1.55625,1.03750)	(2.07500,0.51875)	1.58784	No equilibrium

The Centipede Game theory Figure 7 illustrates a sequential, trust-based interaction between two players over 10 decision-making stages. Each node represents a stage where a player can choose to either cooperate (C) and pass the decision to the next stage, or defect (D) and terminate the game, claiming the current payoff. The game begins at Stage 1 and proceeds rightward, alternating decision points between players. At each stage, the players face the dilemma of trusting the opponent for potentially higher joint payoffs by continuing, or defecting early for immediate personal gain.

The payoffs increase with each cooperative move, reflecting a growing benefit of mutual trust, but also a rising temptation to defect. For example, by Stage 10, the cooperative payoff is (1.56, 1.04) while the defection payoff is (2.08, 0.52), illustrating the imbalance favoring early defection.

However, the absence of equilibrium in all stages indicates persistent strategic tension, with no stable outcome that both players would choose unilaterally. The diagram captures the essence of trust evolution and decision-making dynamics under uncertainty, emphasizing the potential breakdown of cooperation despite mutual gains. It serves as a visual tool to analyze rational behavior in iterated games.

Next, we plot the defective payoffs shown in Figure 8 for both the physician and the nurse over the stages $t = 1$ to $t = 10$, assuming $\delta = 1.1$ and $\beta = 0.8$.

The defective payoffs are given by:

$$(v_t^{(1)}, v_t^{(2)}) = \delta^t \cdot \beta, \delta^t \cdot (1 - \beta))$$

5. Conclusion

This model shows that while classical game theory predicts early defection, introducing trust-based probability functions and collaborative incentive structures can alter the equilibrium and promote sustained cooperation between physicians and nurses in emergency settings. The gap between the Physician's and Nurse's payoffs widens progressively with each stage. This suggests a growing disparity in rewards or benefits over time.

The term defective imply reduced or constrained payoffs due to inefficiencies, misaligned incentives, or systemic issues. Despite this, the Physician's defective payoff still outpaces the Nurse's, possibly highlighting inequities in compensation or recognition in the system. The graph visually demonstrates inequality in growth trajectories between two professional roles in a healthcare setting. It raises questions about fairness, incentive design, and long-term sustainability of such payoff structures. The exponential nature of the curves also suggests that small differences in growth rates can lead to significant disparities over time, warranting strategic adjustments if equity is a goal.

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Theorem 1 (Trust-Modified Equilibrium in Repeated Games). Let

$$\mathrm{EU_C}(\mathrm{t}) = P_C\,(\mathrm{t}) \cdot \delta^{\mathrm{t}} \cdot \alpha + (1 - \mathrm{p_c}\,(\mathrm{t})) \cdot \mathrm{v}_t^{(1)}$$

denote the expected utility of cooperating at stage t in a repeated game, where:

- P_C (t) ∈ [0, 1] is the time-dependent probability of successful cooperation (trust),
- $\delta > 0$ is the discount factor for future rewards, ˆ
- $\alpha > 0$ is the payoff from successful cooperation, ˆ
- $v_t\,(1)$ is the utility from unilateral (non-cooperative) action at stage t.

Then:

$$\mathrm{EU_C}(\mathrm{t}) > \mathrm{v}_t^{(1)} \;\Leftrightarrow\; p_c(t).\left(\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}\right) > 0$$

Therefore, cooperation becomes a rational strategy if and only if

$$\delta^t \cdot \alpha > \mathrm{v}_\mathrm{t}^{(1)} \text{ and } p_c(t) > 0.$$

This implies that:
The existence and evolution of trust, represented by $p_c(t)$, quantitatively shifts the game's equilibrium—rendering cooperation optimal in scenarios where it would not be in the absence of trust.

Corollary 1 (Threshold for Rational Cooperation). Define the trust threshold for rational cooperation as:

$$\mathrm{P_C^{min}}(\mathrm{t}) = \frac{\mathrm{v}_\mathrm{t}^{(1)}}{\delta^{\mathrm{t}} .\ \alpha}$$

Then cooperation is rational **only if**

$$p_c(t) > \mathrm{P_C^{min}}(\mathrm{t})$$

Proof. From the main theorem, we know that cooperation is rational if and only if:

$$\mathrm{EU_C}(\mathrm{t}) > \mathrm{v}_t^{(1)} \;\Leftrightarrow\; p_c(t).\left(\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}\right) > 0$$

Case 1: Suppose $\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}$. Then the term $\left(\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}\right) > 0$, and to preserve the inequality, we must have:

$$p_c(t) > 0$$

We isolate $p_c(t)$ explicitly:

$$p_c(t).\left(\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}\right) > 0 \quad \Rightarrow \quad p_c(t) > \frac{\mathrm{v}_\mathrm{t}^{(1)}}{\delta^{\mathrm{t}} .\ \alpha}$$

This defines the minimum trust threshold:

$$\mathrm{P_C^{min}}(\mathrm{t}) = \frac{\mathrm{v}_\mathrm{t}^{(1)}}{\delta^{\mathrm{t}} .\ \alpha}$$

Case 2: Suppose $\delta^t \cdot \alpha \leq \mathrm{v}_\mathrm{t}^{(1)}$. Then the term $\left(\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}\right) \leq 0$, and the left-hand side becomes:

$$p_c(t).\left(\delta^t \cdot \alpha - \mathrm{v}_\mathrm{t}^{(1)}\right) \leq 0$$

Thus:

$$\mathrm{EU_C}(\mathrm{t}) \leq \mathrm{v}_t^{(1)}$$

and cooperation is not rational in this case. The inequality

$$p_c(t) > \frac{\mathrm{v}_\mathrm{t}^{(1)}}{\delta^{\mathrm{t}} .\ \alpha}$$

has no valid solution because the right-hand side is greater than or equal to 1, while $p_c(t) \leq 1$.

Conclusion: Cooperation is rational if and only if:

$$\delta^t \cdot \alpha > \mathrm{v}_\mathrm{t}^{(1)} \quad \text{and} \quad p_c(t) > \frac{\mathrm{v}_\mathrm{t}^{(1)}}{\delta^{\mathrm{t}} .\ \alpha}$$

which completes the proof.

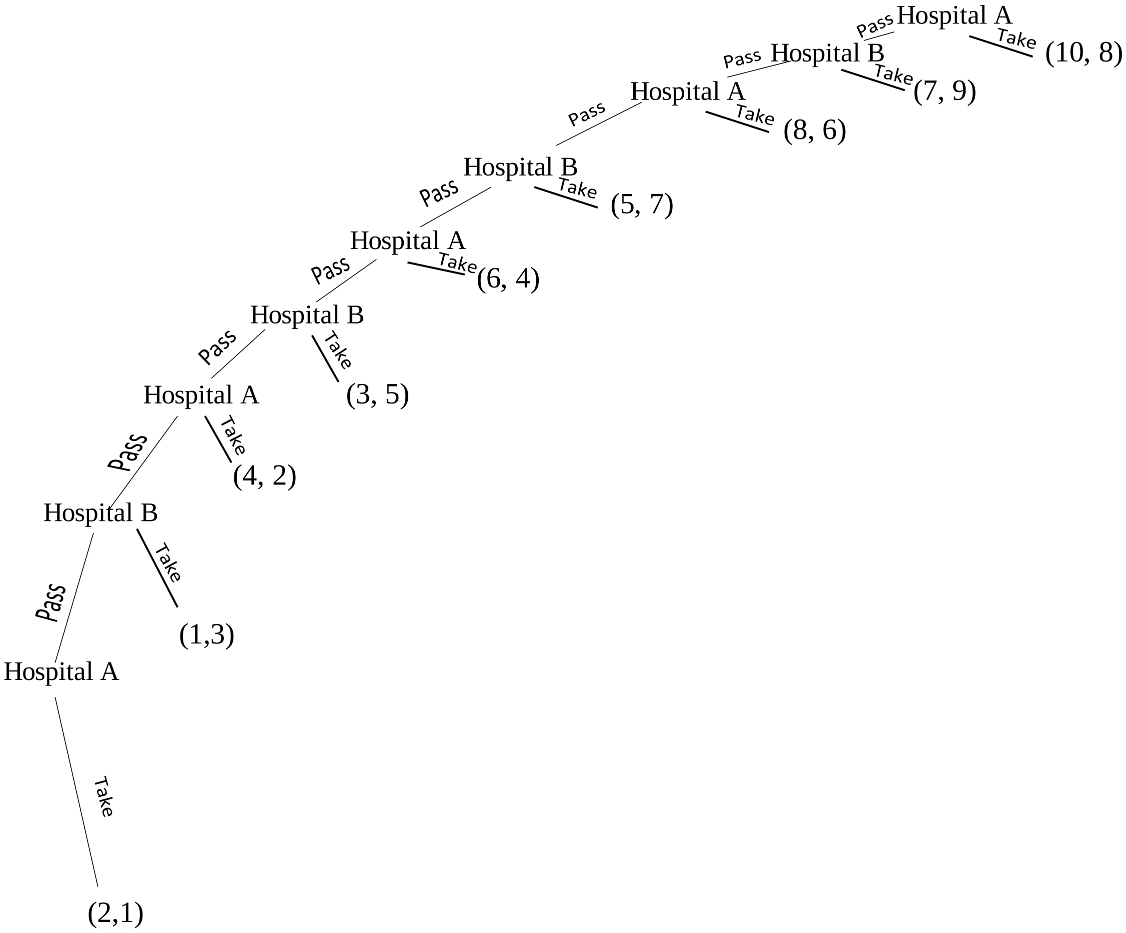

Figure 1: A 10-stage Centipede Game modeling cooperation decisions between two hospitals in healthcare.

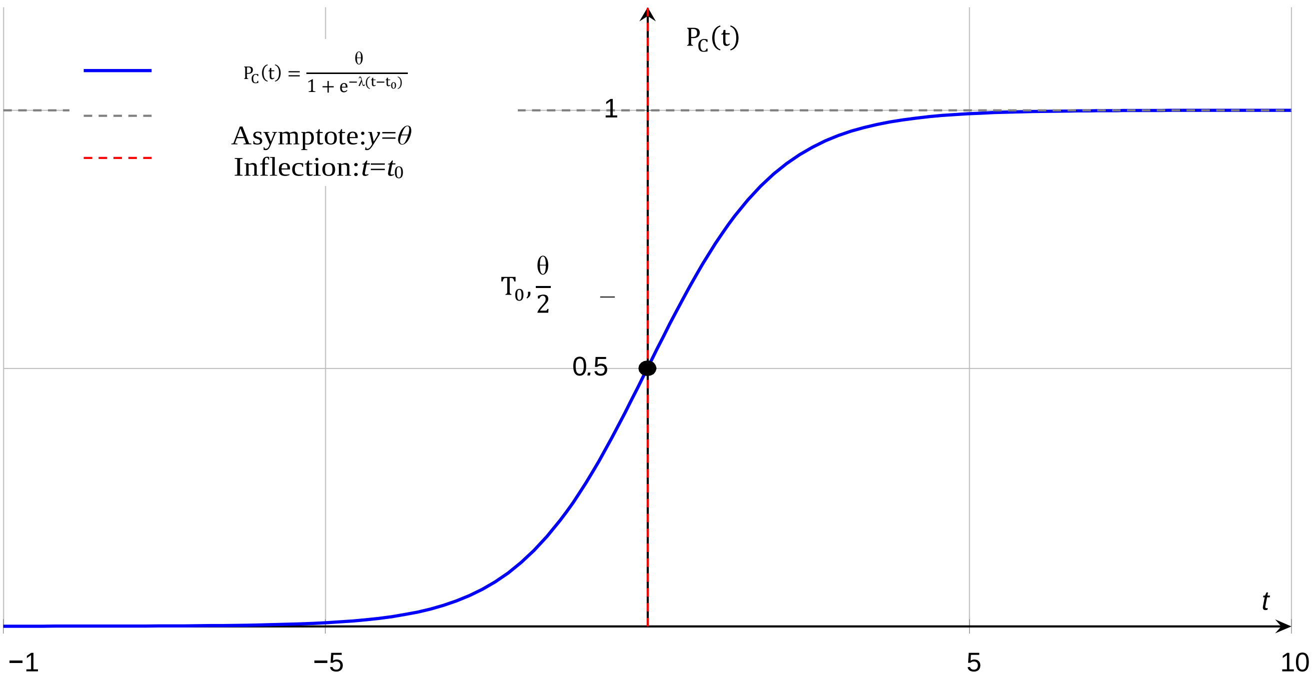

Figure 2: Graph of the sigmoid function $P_C(t) = \frac{\theta}{1+e^{-\lambda(t-t_0)}}$ showing the asymptote at y = θ and inflection point at t = t_0.

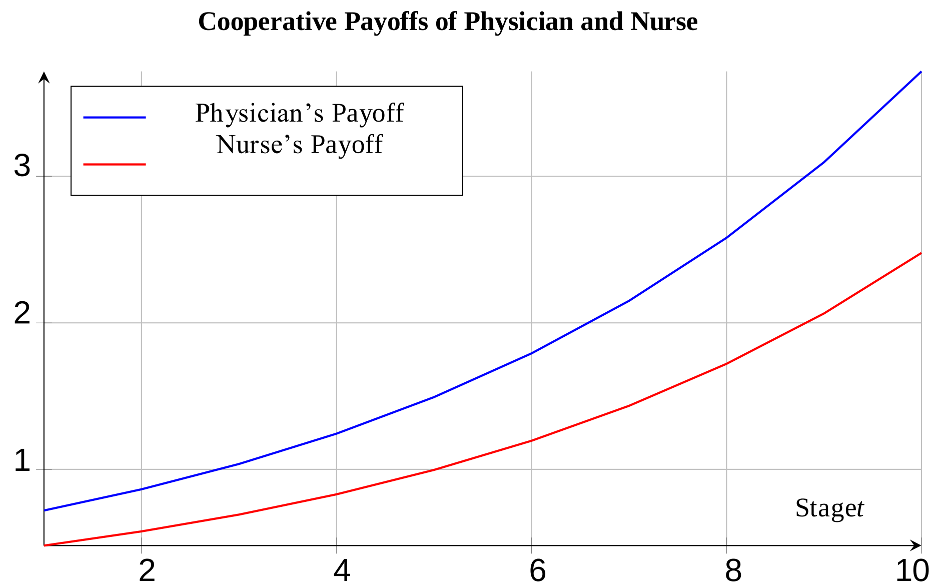

Figure 4: Cooperative Payoffs of Physician and Nurse over 10 stages

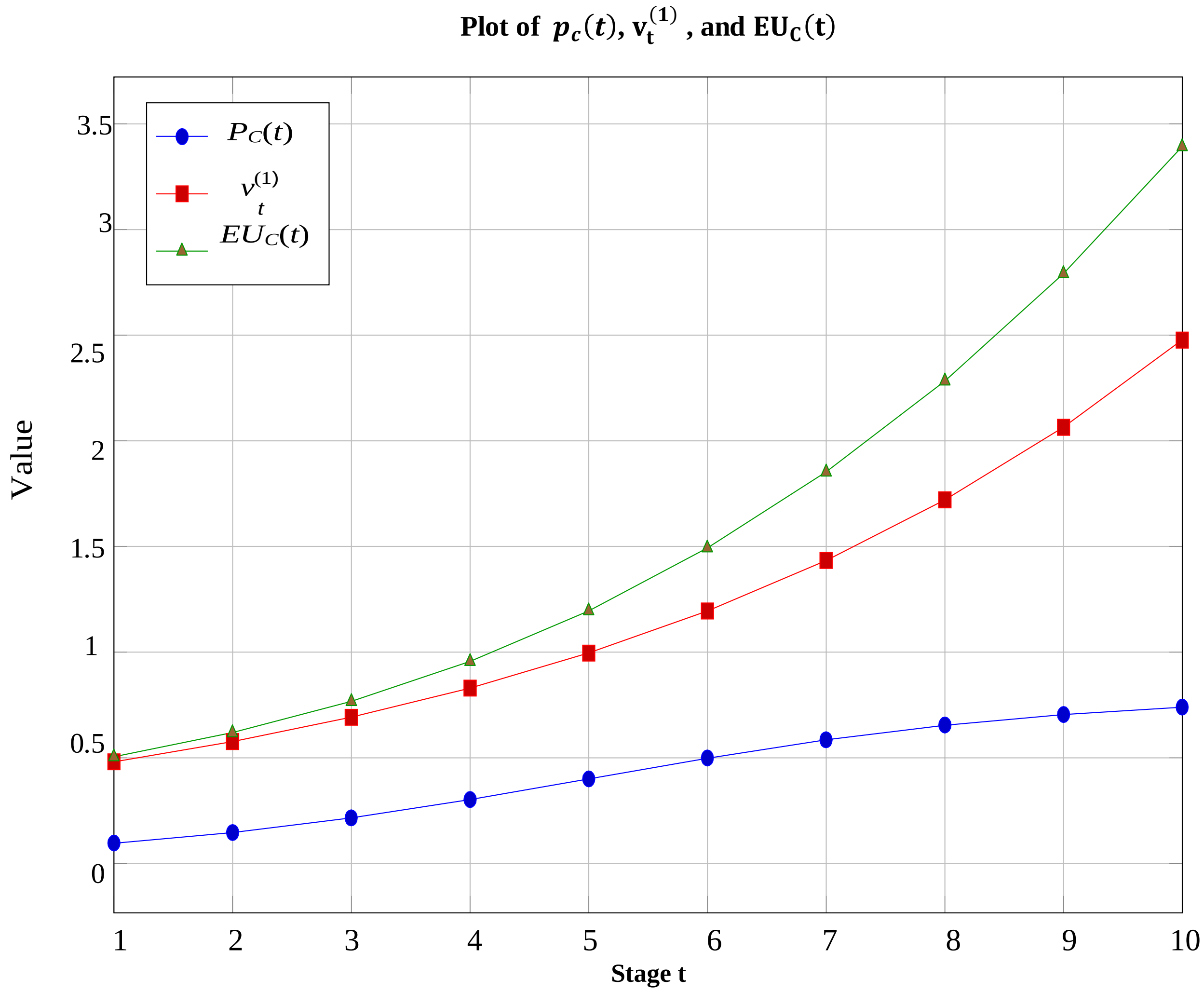

Figure 5: Graph of Cooperation Probability, Value, and Expected Utility

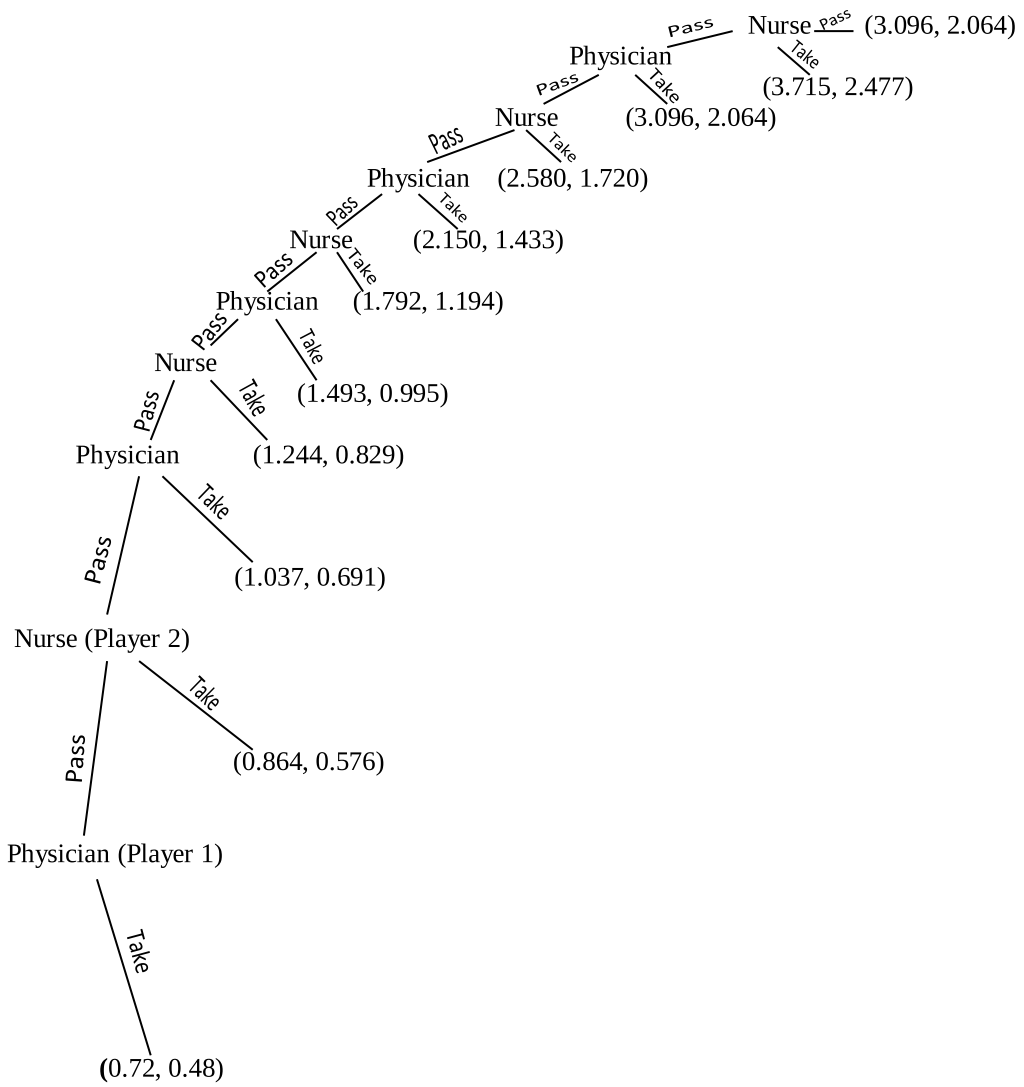

Figure 3: 10-stage Centipede Game in Healthcare: Payoffs for Physician and Nurse at each stage.

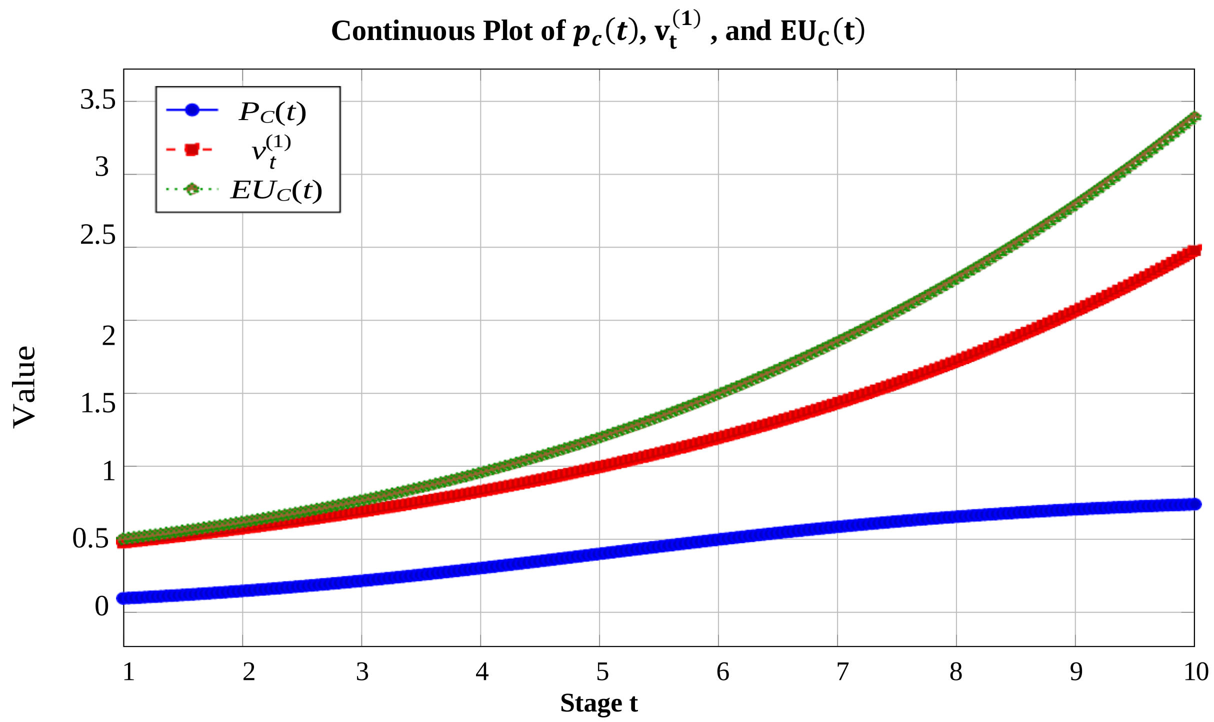

Figure 6: Continuous Graph of Cooperation Probability, Value, and Expected Utility

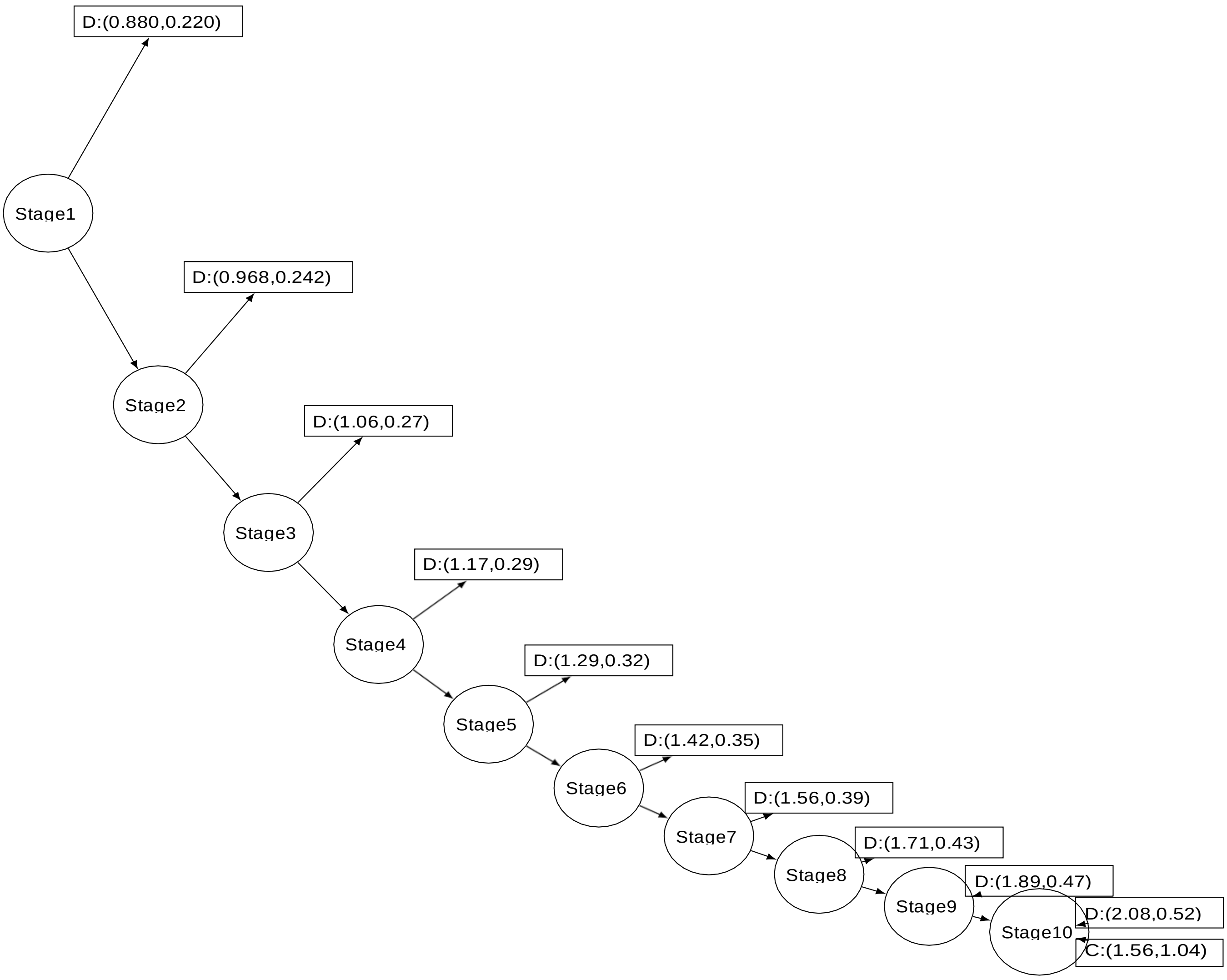

Figure 7: Centipede Game Tree for 10 Stages

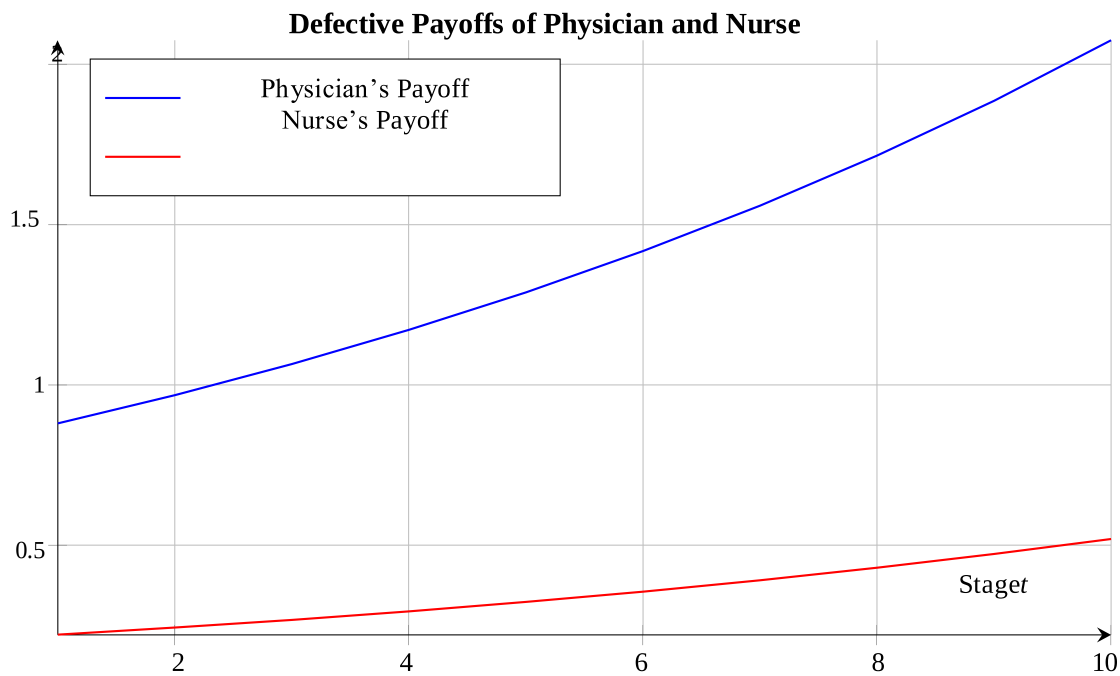

Figure 8: Defective Payoffs of Physician and Nurse over 10 stages