

Comparison of the strength of Modified Extended Median Test for c matched samples and Extended Sign Test by Rankusing a random sample of a certain specie of Bat that were offered dose levels of certain drugs to reduce heartbeat.

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ABSTRACT

Original research paper

Background

Non-parametric tests are those tests normally applied in situations where there are violations of the normality and homogeneity of variance assumptions. In other words, parametric tests cannot be used in such situations. Here, we shall be comparing the statistical power of two non-traditional non-parametric statistical methods suitable for analyzing data that when handled as a two-way Analysis of Variance would be same as a mixed-effects model without replication. They are Extended Sign Test by Ranks for ordered repeated measure and Modified Extended Median Test for c matched samples.

Methods

This paper adopted methods proposed by some authors for Extended Sign Test by Ranks for ordered repeated measure and Modified Extended Median Test for c matched samples. These methods were illustrated using a random sample of a certain specie of Bat offered dose levels of certain drugs to reduce their heartbeat.

Results

On using the method of Extended Sign Test by Ranks for ordered repeated measure to analyze the data, we rejected the null hypothesis of equal dose levels at 5% significant level since $\chi^2 = 11.02 > \chi^2_{0.95;3} = 7.815$.

But at 1% significant level, we accepted the null hypothesis, since $\chi^2 = 11.02 < \chi^2_{0.99;3} = 11.143$. On using Modified Extended Median Test for c Matched samples on data, we rejected the null hypothesis at 5% significant level since $\chi^2 = 25.76 > \chi^2_{0.95;3} = 7.815$ and same also at 1% significant level since $\chi^2 = 25.76 > \chi^2_{0.99;3} = 11.143$.

Conclusions

Extended Sign Test by Ranks for ordered repeated measure has much greater chances of committing Type II Error at 1% significant level, hence Modified Extended Median Test for c Matched samples is more powerful than Extended Sign Test by Ranks for ordered repeated measure and is therefore recommended for use.

Keywords: Extended Sign Test, Modified Extended Median Test, Type II Error, Non-parametric, Dose level.

Introduction

Non-parametric tests are those tests employed as an alternative to parametric tests due to violation of the assumptions of normality and homogeneity of variance. This means that parametric tests cannot be applied without meeting these assumptions. For instance, when the assumptions for the use of Two-Way ANOVA is violated, its non-parametric counterpart is however used as it does not require such strict assumptions. Often times, tests frequently used under these circumstances are Friedman Two-Way ANOVA by Ranks, Cochran's Q test, Modified Extended Median Test, Extended Sign Test by Rank (Oyeka and Okeh, 2013; Oyeka and Umeh, 2015). There are suitable or appropriate for analyzing data that if treated as a Two-Way ANOVA would be the same as a mixed-effects model without replication (Oyeka, 1996). Such data are usually given in tabular form where the column of the table represents one factor having c treatments that are seen as fixed while the row represents another factor having r blocks or levels that are seen as random while each cell has only one observation. These arrangement of the data in a table with c columns and r rows corresponds to the setting seen in a Two-Way ANOVA where as usual one observation appears per cell. As usual for ANOVA, there exists no difference in the c treatments is the null hypothesis to be tested as against the existence of differences in the c treatments for the alternative hypothesis.

In this paper, we shall be comparing the statistical power of two vital non-traditional non-parametric statistical methods that are suitable for analyzing data of the pattern described above. These non-traditional non-parametric procedures are Extended Sign Test by Ranks for ordered repeated measure and Modified Extended Median Test for c Matched samples. The need for this study is to avoid making any wrong decision (accepting false null hypothesis when ordinarily it should be rejected) while using a given non-parametric statistical method in carrying out any test that probably may involve human life.

PROPOSED METHOD 1: Extended Sign Test by Ranks for Ordered Repeated Measure by Oyeka and Umeh (2015).

Oftentimes an investigator may be interested in collecting data from subjects at different times or situations such that there exists repeated measurements from the same set of subjects. Sometimes, these measurements that are in repeated form sampled from same or related populations may not satisfy the needed assumptions required for the application of a suitable parametric test and that will call for the use of its non-parametric counterpart.

In this paper, we apply a proposed method by Oyeka and Umeh (2015) called Extended Sign Test by Ranks for ordered repeated measures. This procedure extended the traditional Sign Test by ranking successive differences between the subjects and making use of their ranks.

According to Oyeka and Umeh (2015), the test statistic is given by

$$\chi^2 = \frac{W_j^2}{Var(W)} = \frac{W^2}{\left(\hat{\pi}^+ + \hat{\pi}^- - (\hat{\pi}^+ - \hat{\pi}^-)^2\right) \sum_{j=1}^{k-1} \sum_{i=1}^n r_{ij}^2} \quad (1)$$

Equation 1 has k-1 Degree of Freedom when the number of observation (n) is reasonably large and under null hypothesis, it is approximately a chi-square distribution.

From Equ.(1),

$$W = (\hat{\pi}^+ - \hat{\pi}^-) \sum_{j=1}^{k-1} R_{.j} = (\hat{\pi}^+ - \hat{\pi}^-) R_{..} = W^+ - W^- \text{ and } R_{..} = \sum_{j=1}^{k-1} R_{.j} = \frac{nk(k-1)}{2}$$

while W^+ and W^- are sums of the ranks of absolute differences between scores with positive and negative signs respectively for the entire subjects and treatment levels. Estimate of sample variance of W in the presence tied observation under null hypothesis is

$$Var(W) = \left(\hat{\pi}^+ + \hat{\pi}^- - (\hat{\pi}^+ - \hat{\pi}^-)^2 \right) \sum_{j=1}^{k-1} \sum_{i=1}^n r_{ij}^2$$

In the absence of tied observations, estimate of sample variance of W is

$$Var(W) = \frac{nk(k-1)(2k-1)}{6} \left(\hat{\pi}^+ + \hat{\pi}^- - (\hat{\pi}^+ - \hat{\pi}^-)^2 \right)$$

Since under the situation $\sum_{j=1}^{k-1} \sum_{i=1}^n r_{ij}^2 = \frac{nk(k-1)(2k-1)}{6}$.

The null hypothesis suitable to be tested here is that the medians across sampled populations (k) are equal.

$$H_0 : \pi_1^+ - \pi_1^- = \pi_2^+ - \pi_2^- = \dots = \pi_k^+ - \pi_k^- = \pi^+ - \pi^- = 0 \text{ versus } H_1 : \pi_j^+ - \pi_j^- \neq 0$$

for some $j = 1, 2, \dots, k-1$.

Null hypothesis is rejected at a given significant level α when $\chi^2 \geq \chi_{1-\alpha; k-1}^2$. If $\chi^2 < \chi_{1-\alpha; k-1}^2$, accept H_0 .

ILLUSTRATIVE EXAMPLE 1

Below in Table 1 shows a randomly sampled population of 17 Bat of certain specie having some percentage reduction in heartbeat resulting from some dose levels of drugs. We here apply the proposed method of Extended Sign Test by Rank from Oyeka and Umeh (2015) to find the differences in the dose levels as shown below.

Table 1: Percentage Reduction in Heartbeat of a Random Sample of 17 Bats of certain specie.

Bat Number (Blocks)	Dose Levels				Differences		
	A	B	C	D	d_{i1}	d_{i2}	d_{i3}
1	3.8	1.6	2.3	3.9	2.2	-0.7	-1.6
2	3.7	3.4	4.5	4.5	0.3	-1.1	0
3	4.2	3.9	4.5	4.2	0.3	-0.6	0.3
4	3.6	2.8	2.6	3.7	0.8	0.2	-1.1
5	3.6	2.2	4.2	3.2	1.4	-2	1
6	2.1	2.8	4.3	4.0	-0.7	-1.5	0.3
7	3.6	2.6	3.2	3.8	1	-0.6	-0.6
8	4.2	2.7	2.9	4.0	1.5	-0.2	-1.1
9	2.9	2.6	3.8	3.6	-0.7	-1	0
10	3.6	3.2	2.7	3.8	0.4	0.5	-1.1
11	3.6	4.5	3.6	3.7	-0.9	0.9	-0.1
12	4.4	3.3	4.1	4.6	1.1	-0.8	-0.5
13	4.1	3.4	4.5	4.3	0.7	-1.1	0.2
14	3.7	3.6	4.2	4.0	0.1	-0.6	0.2
15	3.6	3.1	3.8	4.5	0.5	-0.7	-0.7
16	3.5	3.2	4.0	4.7	0.4	-0.8	-0.7
17	3.8	4.1	4.3	4.8	-0.3	-0.2	-0.5

In order to be able to apply this method effectively such as obtaining the differences d_{ij} and u'_{ij} s, readers are advised to download and study the published work of Oyeka and Umeh (2015).

Table 2. Ranked absolute values of the difference (d_{ij}) and u_{ij} values from Table 1 above.

S/NO	r_{i1}	r_{i1}^2	u_{i1}	$r_{i1}u_{i1}$	r_{i2}	r_{i2}^2	u_{i2}	$r_{i2}u_{i2}$	r_{i3}	r_{i3}^2	u_{i3}	$r_{i3}u_{i3}$	Total
1	3	9	1	3	1.5	2.25	-1	-1.5	3	9	-1	-3	
2	1	1	0	0	2	4	-1	-2	1	1	0	0	
3	1	1	0	0	1.5	2.25	-1	-1.5	1	1	0	0	
4	1.5	2.25	0	0	1	1	0	0	2	4	-1	-2	
5	1.5	2.25	1	1.5	3	9	-1	-3	2	4	1	2	
6	1	1	-1	-1	2	4	-1	-2	1	1	0	0	
7	2	4	1	2	1	1	-1	-1	1	1	-1	-1	
8	1.5	2.25	1	1.5	1	1	-1	-1	2	4	-1	-2	
9	1	1	-1	-1	2	4	-1	-2	1	1	0	0	
10	1	1	0	0	1	1	1	1	3	9	-1	-3	
11	2	4	-1	-2	2	4	1	2	1	1	-1	-1	
12	2	4	1	2	2	4	1	2	1	1	-1	-1	
13	1	1	0	0	2	4	-1	-2	1	1	0	0	
14	1	1	0	0	1	1	-1	-1	1	1	0	0	
15	1	1	0	0	1.5	2.25	-1	-1.5	1.5	2.25	-1	-1.5	
16	1	1	0	0	1.5	2.25	-1	-1.5	1.5	2.25	-1	-1.5	
17	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	
f_j^+			5				2				1		$8(=f^+)$
f_j^0			8				0				6		$14(=f^0)$
f_j^-			4				14				10		$28(=f^-)$
Total n		37.75	17			48	17			43.5	17		51 ($n(k-1)$)
$\hat{\pi}_j^+$			0.1176				0.1176				0.059		$0.16(\hat{\pi}^+)$
$\hat{\pi}_j^0$			0.4705				0				0.353		$0.27(\hat{\pi}^0)$
$\hat{\pi}_j^-$			0.2352				0.8235				0.588		$0.55(\hat{\pi}^-)$
R_j	24				27				25				$76(=R)$
W_j				5				-20				-13	$-28(=W)$

Values of f_j^+ , f_j^0 , f_j^- , $\hat{\pi}_j^+$, $\hat{\pi}_j^0$, $\hat{\pi}_j^-$, R_j , and W_j are calculated as explained above and shown in Table 2.

Since $W = \sum_{j=1}^{k-1} (\pi_j^+ - \pi_j^-) R_j = W^+ - W^-$, we substitute accordingly,

$$W = (0.1176 - 0.2352)(24) + (0.1176 - 0.8235)(27) + (0.059 - 0.588)(25) \\ = -2.8224 - 19.0593 - 13.225 = -35.1067$$

From equation 17, we have that

$$\text{Var}(W) = (0.1176 + 0.2352 - (0.1176 - 0.2352)^2)(37.75) + (0.1176 + 0.8235 - (0.1176 - 0.8235)^2)(48) \\ + (0.059 + 0.588 - (0.059 - 0.588)^2)(43.5) = (0.3528 - 0.01382976)(37.75) + (0.9411 - 0.49829481)(48) \\ + (0.647 - 0.279841)(43.5) = 12.79613 + 21.25465 + 15.9714 = 50.02218.$$

Hence,

$$\chi^2 = \frac{(-35.1067)^2}{50.02218} = 24.639.$$

The chi-square has 4-1=3 degrees of freedom and based on this, we reject the null hypothesis and conclude that there exists statistically significant relationship at 5% level of significance since $\chi^2_{0.95;3} = 7.815$. We estimate

$$\hat{\pi}^+ = \sum_{j=1}^{k-1} \frac{\hat{\pi}_j^+}{k-1} = \sum_{j=1}^{k-1} \frac{f_j^+}{n(k-1)}; \hat{\pi}^0 = \sum_{j=1}^{k-1} \frac{\hat{\pi}_j^0}{k-1} = \sum_{j=1}^{k-1} \frac{f_j^0}{n(k-1)}; \hat{\pi}^- = \sum_{j=1}^{k-1} \frac{\hat{\pi}_j^-}{k-1} = \sum_{j=1}^{k-1} \frac{f_j^-}{n(k-1)}$$

using the frequencies in Table 2 as

$$\hat{\pi}^+ = \frac{8}{51} = 0.156; \hat{\pi}^0 = \frac{14}{51} = 0.2745; \hat{\pi}^- = \frac{28}{51} = 0.549.$$

Also from Table 2, we have that $W = W_1 + W_2 + W_3 = 5 + (20) + (13) = -28$.

Given the presence of ties,

$$Var(W) = \left(\hat{\pi}^+ + \hat{\pi}^- - \left(\hat{\pi}^+ - \hat{\pi}^- \right)^2 \right) \sum_{j=1}^{k-1} \sum_{i=1}^n r_{ij}^2 \text{ as}$$

$$Var(W) = (0.156 + 0.549 - (0.156 - 0.549)^2)(37.75 + 48 + 43.5) = (0.705 - 0.15449)(129.25) = 71.15.$$

We therefore test for the equality of medians for the four populations as,

$$\chi^2 = \frac{(-28)^2}{71.15} = 11.02.$$

The chi-square has 4-1=3 degrees of freedom and since $\chi^2 = 11.02 > \chi^2_{0.95;3} = 7.815$, we therefore reject H_0 of equal dose levels at 5% level of significant and conclude that significant relationship exists. In other words, the response between sample dose levels of certain drugs of a randomly selected specie of Bat are not consistent with one another. This also implies that the response between sample dose levels of the drugs are significantly different. But at 1% significant level, we accept the null hypothesis of equality of sample dose levels, thus the chances of committing Type II Error is very high since $\chi^2 = 11.02 < \chi^2_{0.99;3} = 11.143$.

PROPOSED METHOD 2: Modified Extended Median Test for c Matched samples by Oyeka and Okeh (2013).

The Extended Median Test is suitable for analyzing matched data or samples. Measurements here may not necessarily be numeric or continuous but can be on ordinal scale before samples are drawn from the population of interest (Gibbon, 1971, 1993). As usual data are similarly presented in table form that is also suitable for analyzing Two-Way ANOVA. The null hypothesis usually tested here are also same with that for analyzing data using Extended Sign Test by Rank (Agresti, 1992).

According to Oyeka and Okeh (2012, 2013), this proposed method can also be suitably utilized for analyzing data having the same pattern as when Friedman test is carried out. Just like when Extended Sign Test by Rank was applied to the data, in this proposed method, it is also required to find the median for the 'c' treatments or populations and similar test of hypothesis is applied here also.

However, for better understanding of this proposed method, readers are urged to download the paper and understand the pattern as that is not the focus of this work.

ILLUSTRATIVE EXAMPLE 2

Table 3: Percentage Reduction in Heartbeat of a Random Sample of 17 Bats of certain specie.

Bat (Blocks)	Number	Dose Levels			
		A	B	C	D
1		3.8	1.6	2.3	3.9

2	3.7	3.4	4.5	4.5
3	4.2	3.9	4.5	4.2
4	3.6	2.8	2.6	3.7
5	3.6	2.2	4.2	3.2
6	2.1	2.8	4.3	4.0
7	3.6	2.6	3.2	3.8
8	4.2	2.7	2.9	4.0
9	2.9	2.6	3.8	3.6
10	3.6	3.2	2.7	3.8
11	3.6	4.5	3.6	3.7
12	4.4	3.3	4.1	4.6
13	4.1	3.4	4.5	4.3
14	3.7	3.6	4.2	4.0
15	3.6	3.1	3.8	4.5
16	3.5	3.2	4.0	4.7
17	3.8	4.1	4.3	4.8

Applying the procedures clearly described by Oyeka and Okeh (2013) on the data of Table 3 above, we obtain Table 4.

Table 4: Block Median, values of u_{ij} and order statistics for the data of Table 3

Bat Number (Blocks)	Block Medians (M_i)	Dose Levels				
		A	B	C	D	
1	3.05	1	-1	-1	1	
2	4.1	-1	-1	1	1	
3	4.2	0	-1	1	0	
4	3.2	1	0	-1	1	
5	3.4	1	-1	1	-1	
6	3.2	-1	-1	1	1	
7	3.4	1	-1	-1	1	
8	3.45	1	-1	-1	1	
9	3.4	-1	-1	1	1	
10	3.4	1	-1	-1	1	
11	3.65	-1	1	-1	1	
12	4.25	1	-1	-1	1	
13	4.2	-1	-1	1	1	

14	3.85	-1	-1	1	1	
15	3.7	-1	-1	1	1	
16	3.75	-1	-1	1	1	
17	4.2	-1	-1	1	1	
Total(k)		17	17	17	17	68(= kc)
f_j^+		7	1	10	15	33(= f^+)
f_j^-		9	15	7	1	32(= f^-)
f_j^0		1	1	0	1	3(= f^0)
P_j^+		0.4118	0.0588	0.5882	0.8823	0.4853(P^+)
P_j^-		0.5294	0.8823	0.4118	0.0588	0.4706(P^-)
P_j^0		0.0588	0.0588	0.0	0.0588	0.0441(P^0)

This method proposed by Oyeka and Okeh (2013), has been applied on the data and the results are shown in Table 4 above. The test statistic is given as

$$\chi^2 = \frac{k}{P^+ P^- (1 - P^+ - P^-)} \left(P^- (1 - P^-) \left(\sum_{j=1}^c P_j^{+2} - c P^{+2} \right) + P^+ (1 - P^+) \left(\sum_{j=1}^c P_j^{-2} - c P^{-2} \right) + 2 P^+ P^- \left(\sum_{j=1}^c P_j^+ P_j^- - c P^+ P^- \right) \right) \quad (2)$$

Equation 2 has 2(c-1) degrees of freedom and is a chi-square distribution. Hence,

$$\begin{aligned} \chi^2 &= \frac{17}{(0.4853)(0.4706)(0.0441)} \left((0.4706)(1 - 0.4706) [0.4118^2 + 0.0588^2 + 0.5882^2 + 0.8823^2 - 4 \times 0.4853^2] \right. \\ &= + (0.4853)(1 - 0.4853) [0.5294^2 + 0.8823^2 + 0.4118^2 + 0.0588^2 - 4 \times 0.4706^2] \\ &+ (2)(0.4853)(0.4706) [0.4118 \times 0.5294 + 0.0588 \times 0.8823 + 0.5882 \times 0.4118 + 0.8823 \times 0.0588 - 4 \times 0.4853 \times 0.4706] \end{aligned}$$

$$\begin{aligned} \chi^2 &= \frac{17}{0.01007} (0.2491(0.1696 + 0.00346 + 0.34598 + 0.7785 - 0.9421) + \\ &0.2498[0.2803 + 0.7785 + 0.1696 + 0.00346 - 0.8859] + 0.4568 \\ &[0.2180 + 0.05188 + 0.2422 + 0.0518 - 0.91353]) \end{aligned}$$

$$\chi^2 = \frac{17}{0.01007} [0.08854 + 0.08642 + (-0.1597)]$$

$$\chi^2 = \frac{17}{0.01007} 0.01526 = \frac{0.25942}{0.01007} = 25.76$$

$$\chi^2 = 25.76.$$

This calculated chi-square has 2(c-1)=6 degrees of freedom and since $\chi^2 = 25.76 > \chi_{0.95;3}^2 = 7.815$, we reject H_0 of equality of dose levels of drugs for the randomly selected Bats and conclude that there exists a statistically significant relationship, meaning that the response between sample dose levels of certain drugs for the randomly selected species of Bat are not consistent with one another. At 1% significant level, we also found that the null hypothesis (H_0) is rejected

$$\chi^2 = 25.76 > \chi_{0.99;3}^2 = 11.143.$$

Summary and Conclusion

We conclude that even though the analysis of the data using the two methods, namely, Extended Sign Test by Ranks for ordered repeated measure and Modified Extended Median Test for c Matched samples lead to rejection of H_0 of equality of dose levels of the percentage reduction in heartbeat of certain specie of bat at 5% significant levels, at 1% significant level, the earlier method (Extended Sign Test by Ranks for ordered repeated measure) witnessed the acceptance of false null hypothesis, thus having greater chance of committing Type II Error. In conclusion, we say that Modified Extended Median test for c Matched samples is less likely to lead to an acceptance of a false null hypothesis (Type II Error) more frequently and hence likely to be more powerful than the Extended Sign Test by Ranks for ordered repeated measure. Therefore, Modified Extended Median Test for c Matched samples is nevertheless recommended for determining the consistency of multiple response samples, a function

usually performed by the traditional Friedman's two-way ANOVA by Ranks.

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