IKR Publishers [International Knowledge and Research Publishers]

IKR Journal of Clinical Medicine and Medical Research (IKRJCMMR)

Journal homepage: https://ikrpublishers.com/ikrjcmmr/

Volume-1, Issue-1 (May-June) 2025

ISSN: 3107-6521 (Online)

Assessing Cochran's Q Test and Pairwise Extended McNemar Test for ProportionalConsistency in a Dichotomous Nominal Scale Data

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ARTICLE INFO

Article history:

Received : 12-05-2025

Accepted : 14-05-2025

Available online : 16-05-2025

Keywords:

Extended McNemar test, Type I Error, Cochran's Q test, drug preparation data, improved, non-improved, control, dichotomous.

ABSTRACT *Original research paper*

Background

The Cochran's Q test is a statistical procedure for assessing the conformity of proportions for various samples in dichotomous outcomes. This process extends from the traditional McNemar test for paired data.

Method

In other to use Cochran's Q test, paired and independent samples are assumed while the outcome variables are dichotomous also. This test calculates the Q test statistic, which calculatesunder the null hypothesis the difference between the observed and expected frequencies. The test statistic is then compared to a Chi-squared distribution to determine the p-value. In order to control for Type I Error, the study employed extended McNemar test from a study carried by Okeh and Obiora-Ilouno. These drug preparation samples were paired and a comparison study was carried out using the extended McNemar tests.

Results

Cochran's Q test was applied on the drug preparation data and result concluded the acceptance of H_0 of equality of drug samples. We conducted a comparison of the paired samples to determine the contributions of various pairs of the sample data using extended McNemar tests. Results showed that patients did not improve equally for some of the pairs of drugs preparations. This is a deviation from the result obtained when Cochran's Q test was applied on the data.

Conclusion

The use of extended McNemar test for the pairwise comparison of the data clearly identified sample pairs which their contributions would have made the study to suggest the rejection of H_0 of equally of drug preparations, thus controlling for Type I Error. This is particularly a significant role usually played by McNemar test.

INTRODUCTION

The Cochran's Q test for dichotomous nominal scale data is a statistical method used to evaluate the steadfastness or uniformity of proportions in a multiple responsedataset that are dichotomous in nature.Most often, they are used to analyze data that are categorically arranged in an n × k contingency table, where n and k stands for the numbers of observations and groups that are compared respectively. The Cochran's Q test aims to find out if the proportions of a particularresponse which may be present or absent, yes or no are persistence across the multiple response groups. The null hypothesis usually applied in this test is that all the multiple responses acrossgroups are the same against the alternative hypothesis that some response(s) in the group differs in theirproportion. Because Cochran's Q test statistic helps to verify the existence of any significant differencesin proportions across response categories, it is an important method used in many areas in medical field, physical sciences, and other research areas where it is vital to analyze categoricaldata with a view to identifying their patterns or associations. Song and Wasselcame up with a formula for finding the sample size of Cochran's Q test and this formulainvolved an added information on stratum-specificsuccess rates [1]. Similarly,Kulinskaya and Dollingerpointed out the shortcomings in the use of chi-square test statisticin place of Cochran's Q statistic in the testof homogeneity in meta-analysisgiven that the effects of the studies of odds ratios are in logarithms [2]. Okeh et al.proposeda statistical procedure for analyzing sample data with mainly dichotomousresponses and considering only two mutually exclusive values [3]. Stephen and SAZexplored the practical application of Cochran's Q test and pairwise McNemar testto examine the proportion of responses derived from the results of Multiple Responses Analysis (MRA) [4].Van Aert et al. showed confidence intervals as well as point estimates of between-study variance and these aided the interpretation of meta-analytic results [5]. The Q-profile and generalized Q-statistic are however two methodsthat usually utilize the Q-statistic in creatingthese confidence intervals. Some illustrative examples of Cochran's Q test statistic were shown by some authors [6],[3].

PROPOSED METHOD

Sometimes a researcher may perform an experiment involving repeated observations,or block,in which the variable of interest is dichotomous,meaning that it can assume only one of two possible values.One of these two values is considered a 'success',often coded with a '1', and the other is considered a 'failure',often coded with a'0'.The researcher's interest may be to determine if the rightprobability of successes is the same for the experimental subjects across the treatment levels. The appropriate test to be applied in a situation like this is the Cochran's Q test for binary outcomes.In other to use the Cochran's Q test,data are placed in tabular form wherethe column represents the c treatments and the row designating the r subjects or blocks.Then the entry in the cell formed by the ith row where i=1,2,…,r and jth column where j=1,2,….,c is coded 1,if it is a success,0,if it is a failure.Finally,we find the sum of the number of 1's in ith row,that is,block i. This is designated by B. Similarly,we find the sum of the number of 1's in jth column,that is treatment j. This is designated by T_j.

Cochran's Q test statistic is given is

$$Q=(c-1)\frac{\left[\sum_{j=1}^{c}T_j^2-\left(\sum_{j=1}^{C}T_j^2\right)^2\Big/c\right]}{\sum_{i=1}^{r}B_i-\left(\sum_{j=1}^{c}B^2\right)\Big/c} \qquad (1)$$

The usual null hypothesis to be tested here is that probability of success areequal for all the c treatments so that the Q test statistic is distributed approximately as Chi-square with c-1 degrees of freedom. This can be compared with an appropriate critical or tabulated Chi-square value to decide whether to reject or accept the null hypothesis of equality of probability of success.

ILLUSTRATIVE EXAMPLE

Patients reactions after using four drugs were studied so as to determine at 1 percentage significant level if the four drugs preparations improved patient's health conditionequally.A total of 45 patients were sampled in such a way that 15 blocks were obtained with four patients in each block almost identical in age,initial condition,sex,and in other characteristics.By randomization, patients for each block were selected for treatment so that only one of the four experimental drugs were given.At the end of the period of drugs administration,the patients were grouped as improved or success and coded as 1 or not improved or failure and coded as o,respectively.Below is the data for illustrating the Cochran's Q Test.

Table 1: The response of patients to four drug preparations where improved=1 and Not improved=0.

Patients(Blocks)	**Four drug preparations**				Number of improved
	Diet 1	Diet 2	Diet 3	Diet 4	
1	1	1	0	0	2
2	1	1	0	1	3
3	1	0	0	0	1
4	1	1	1	1	4
5	1	1	0	1	3

6	0	1	0	0	1
7	0	1	1	0	2
8	1	1	1	0	3
9	0	0	1	0	1
10	0	0	0	0	0
11	0	0	0	0	0
12	1	1	0	1	3
13	1	0	0	1	2
14	0	1	1	0	2
15	1	1	0	0	2

Here the null hypothesis required is that patient improvement was the same on the four drugs.The alternative hypothesis is that patients did not improve equally on the four drugs preparations.The null hypothesis would be tested using the Cochran'Q test. Substituting in the formula,we have

$$Q=\frac{(4-1)(10)+(10)^2+(6)^2+(5)^2-(10+6+6+5)^2/4}{31-(2+3+....+2)-\left((2)^2+(3)^2+....+(2)^2\right)/4}$$

$$=\frac{(3)(261-240.25)}{31-19.75}=\frac{62.25}{11.25}=5.53$$

which has c-1 =4-1=3 degrees of freedom. Since Q=5.33 is less than $11.35=\chi^2_{0.99;3}$, we accept the null hypothesis of equal improvement of patients across treatment groups. In conclusion, patients improved equally on the four drugs preparations.

Extended McNemar Test for Controlling Type I Error rate

When carrying out Cochran's Q test on multiple response data that are dichotomous which is a method of conducting pairwise comparisons, often times the risk of Type I errors (false positives) increases.By using traditional McNemar test, the Type I error rate for each pair samplecan be controlled,which shows reliable results.According to Okeh and Obiora-Ilouno, traditional McNemar testis limitedby the fact that there exists ties in the data usually used because the data in most cases are quantitative in nature and may also be continuous[7]. We here apply extended McNemar test based on the work carried out by Sumi et al, [8] who adopted the traditionalMcNemar test. This application allows the possible presence of ties in the data. By adopting the procedures of extended McNemar test [7], we present a fourfold table of paired sample data as

Table 2. Fourfold Table for Paired Comparison of Multiple Response Dichotomous Data

	Sample 2		
Sample 1	**Success**	**Failure**	**Total**
Success	$n_{11}=p^{0+}$	$n_{12}=p^{+}$	$n_{11}+n_{12}$
Failure	$n_{21}=p^{-}$	$n_{22}=p^{0-}$	$n_{21}+n_{22}$
Total	$n_{11}+n_{21}$	$n_{12}+n_{22}$	$n_{..}(=N)$

In line with the results obtained [7],the null hypothesis that is usually applied here in paired comparison testing is

$$H_0:\pi^+-\pi^-=\beta_0 \text{ versus } H_0:\pi^+-\pi^-\neq\beta_0\ (-1\le\beta_0\le 1)$$

While the test statistic is given by

$$\chi^2=\frac{n\left(\left(\hat{\pi}^+-\hat{\pi}^-\right)-\beta_0\right)^2}{\hat{\pi}^+-\hat{\pi}^--\left(\hat{\pi}^+-\hat{\pi}^-\right)^2} \qquad (2)$$

Which with 1 degree of freedom is approximately chi-square distributed for sufficiently large n.

Here

$$P_1=\frac{n_{11}+n_{21}}{N}=\frac{p^{0+}+p^{-}}{N}=\hat{\pi}^{0+}+\hat{\pi}^{-} \text{ and } P_2=\frac{n_{11}+n_{12}}{N}=\frac{p^{0+}+p^{+}}{N}=\hat{\pi}^{0+}+\hat{\pi}^{+}$$

$$\text{where } \hat{\pi}^{0+}=\frac{p^{0+}}{N}$$

$$\text{and } \hat{\pi}^{0-}=\frac{p^{0-}}{N} \text{ such that } \hat{\pi}^{0}=\hat{\pi}^{+}+\hat{\pi}^{-}. \text{ If } \chi^2\ge\chi^2_{1-\alpha;1}$$

, we reject the null hypothesis that the proportion of treatment groups are equal at the 5 % level of significance which concludes that proportion of treatment groups are not equal.

In other to control for Type I Error that may have occurred while applying Cochran's Q test, on the data for finding the consistency in the proportion of patients' drug preparations,we pair the four drug preparations samples and carry out extended McNemar tests on all of them. The pairing of the diets is in the following order:1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4, as well as 3 and 4. Based on these paired arrangement, 6 fourfold classificationtables will emerge from the existing Table 1 and extended McNemar tests will be carried on all of them at 5% level of significance. These tables are as follows:

Table 2: Fourfold Classification of Diet 1 and 2 Drug Preparations

	Patients' Response in Diet 1		
Patients' Response in Diet 2	Improved	Non-improved	Total
Improved	7	3	10
Non-improved	2	3	5
Total	9	6	30

$\chi^2=4.602, \chi^2_{0.95;1}=3.841.$ *Here Ho is rejected.*

Table 3: Fourfold Classification of Diet 1 and 3 Drug Preparations

	Patients' Response in Diet 1		
Patients' Response in Diet 3	Improved	Non-improved	Total
Improved	2	3	5
Non-improved	7	3	10
Total	9	6	30

$\chi^2 = -0.9584, \chi^2_{0.95;1} = 3.841.$ *Here Ho is accepted.*

Table 4: Fourfold Classification of Diet 1 and 4 Drug Preparations

	Patients' Response in Diet 1		
Patients' Response in Diet 4	Improved	Non-improved	Total
Improved	5	0	5
Non-improved	4	6	10
Total	9	6	30

$\chi^2 = -0.9584, \chi^2_{0.95;1} = 3.841.$ *Here Ho is accepted.*

Table 5: Fourfold Classification of Diet 2 and 3 Drug Preparations

	Patients' Response in Diet 2		
Patients' Response in Diet 3	Improved	Non-improved	Total
Improved	4	1	5
Non-improved	6	4	10
Total	10	5	30

$\chi^2 = 30, \chi^2_{0.95;1} = 3.841.$ *Here Ho is rejected*

Table 6: Fourfold Classification of Diet 2 and 4 Drug Preparations

	Patients' Response in Diet 2		
Patients' Response in Diet 3	Improved	Non-improved	Total
Improved	4	1	5
Non-improved	6	4	10
Total	10	5	30

$\chi^2 = 30, \chi^2_{0.95;1} = 3.841.$ *Here Ho is rejected*

Table 7: Fourfold Classification of Diet 3 and 4 Drug Preparations

	Patients' Response in Diet 3		
Patients' Response in Diet 4	Improved	Non-improved	Total
Improved	1	4	5
Non-improved	4	6	10
Total	5	10	30

$\chi^2 = -4.293, \chi^2_{0.95;1} = 3.841.$ *Here Ho is accepted.*

Results obtained from the analysis of these pairs (Tables 2 to 7) of the four drug preparations samples in order to control for Type I Error that may have occurred while applying Cochran's Q test on the data are stated below: Association between Diets 1 and 2 Drug Preparations resulted in the rejection of H0.Association between Diets 1 and 3 Drug Preparations resulted in the acceptance of H_0. Association between Diets 1 and 4 Drug Preparations resulted in the acceptance of H_0. Association between Diets 2 and 3 Drug Preparations resulted in the rejection of H0. Association between Diets 2 and 4 Drug Preparations resulted in the rejection of H_0. Association between Diets 3 and 4 Drug Preparations resulted in the acceptance of H_0.

Results therefore showed that patients did not improve equally for the pairs of drugs preparations where the conclusion suggested rejecting the H_0 which is actually a deviation from the result obtained when Cochran 's Q test was applied on the data. By these analysis, the use of extended McNemar test for the pairwise comparison of the data has clearly identified sample pairs which their contributions would have made the study to suggest the rejection of H_0 of equally of drug preparations, thus controllingfor Type I Error.This is particularly a significant role usually played by McNemar test.

Summary and conclusions

Results therefore showed that patients did not improve equally for the pairs of drugs preparations where the conclusion suggested rejecting the H_0 which is actually a deviation from the result obtained when Cochran 's Q test was applied on the data. By these analysis, the use of extended McNemar test for the pairwise comparison of the data has clearly identified sample pairs which their contributions would have made the study to suggest the rejection of H_0 of equally of drug preparations, thus controlling for Type I Error. This is particularly a significant role usually played by McNemar test. This study clearly showed that Cochran's Q test are usually used to indicate if there exists any difference significantly between three or more related samples with dichotomous outcomes.Here in this study,four drug preparation was a case study. However, Cochran's Q test does not specify which pairs of samples differ significantly. In this study, extended McNemar pairwise comparison tests helped to identify the specific pairs of samples that contribute to the significant overall effect.When conducting multiple pairwise comparisons, the risk of type I errors (false positives) increases. By using extendedMcNemar tests, this study controlled the Type I Error rate for comparing each sample pair which therefore provides reliable results. The use of extended McNemar tests accounted for the correlation between drug preparation paired

data. This is actually essential when working with multiple responses sample data like these four drug preparation data. This ensures that the test results accurately reflect the relationships between the samples. The results obtained from the use of extended McNemar pairwise comparison tests provided more detailed insights into the relationships between the drug samples. By examining the results of each pairwise comparison, one will have a better understanding of the underlying patterns and differences in the drug sample. The results of these extended McNemar pairwise comparison tests can inform future research or decision-making. By identifying specific areas of difference or similarity in these drug preparation, one can refine his/her research questions, develop more targeted interventions, or make more informed decisions.

Acknowledgement

We wish to thank the staff of medical record unit of the Federal University Hospital Abakaliki Nigeria where data was collected for their effort in making the data used for this work available.

Declarations

No ethics process was engaged.

Conflict of interest

None.

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